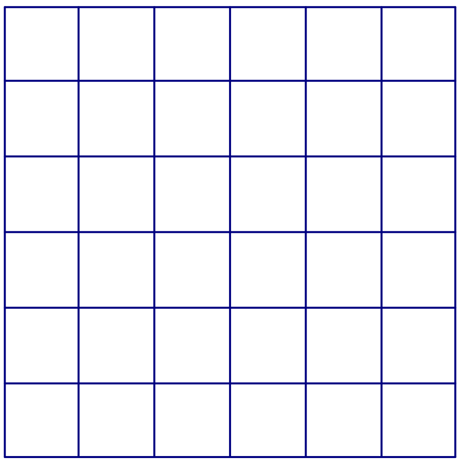
MULTIPLYING FRACTIONS Name ___________________________________

Earlier in the quarter we looked at a rectangle model for multiplication of whole numbers. Assuming that each small square in the grid below is 1 square unit, sketch a picture that represents 3 X 4. Label both dimensions.

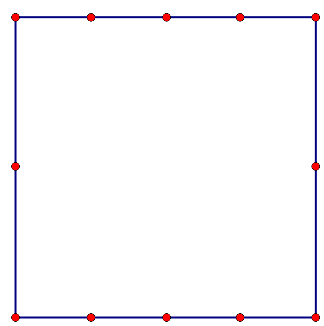
The answer to the multiplication problem 3 X 4 is found by counting the squares contained within that rectangle. Another way to think of this is to say that the answer is the area contained within a 3 by 4 rectangle.



This is a useful model not only for showing whole number multiplication, but also for showing multiplication of fractions.

If you want to model a problem like $\frac{1}{2}\cdot\frac{3}{4}$ you can begin with a single rectangle that represents 1 unit. By making a rectangle that is $\frac{1}{2}$ by $\frac{3}{4}$ and finding its area, we are finding the product of $\frac{1}{2}\cdot\frac{3}{4}$.

The "unit" rectangle to the right has been pre-marked to make dividing the rectangle easier.

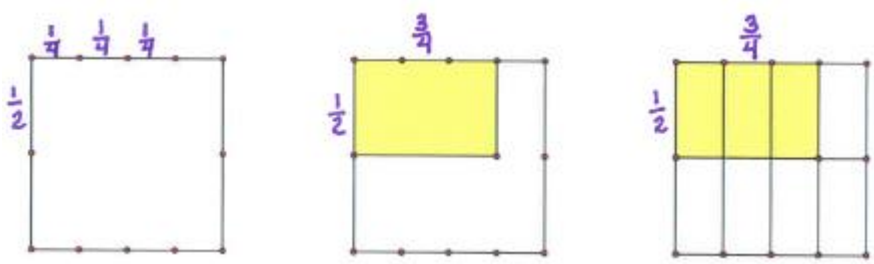


Use the "unit" rectangle to make a smaller rectangle with one dimension being $\frac{1}{2}$ and the other dimension being $\frac{3}{4}$.

Label both dimensions and shade in the $\frac{1}{2}$ by $\frac{3}{4}$ rectangle.

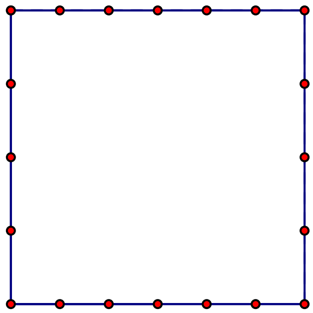
See the progression of doing this below.

What fraction of the original rectangle is shaded in?



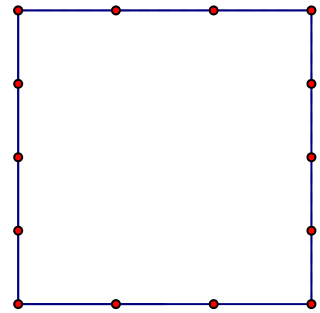
The shaded rectangle has dimensions $\frac{1}{2}$ by $\frac{3}{4}$. Since $\frac{3}{8}$ of the unit rectangle is shaded, $\frac{1}{2}\cdot\frac{3}{4}=\frac{3}{8}$.

Repeat the process for $\frac{3}{4}\cdot\frac{5}{6}$



$\frac{3}{4}\cdot\frac{5}{6}=$

Repeat once again for $\frac{1}{4}\cdot\frac{2}{3}$



$\frac{1}{4}\cdot\frac{2}{3}=$

You should be noticing that you are able to find the product of two proper fractions by multiplying the numerators together (this is the shaded area) and the denominators together (this is the total number of subsections in the whole). This is important, because you don't want to have to draw rectangles every time you want to multiply fractions.

Describe how you can get the answer for $\frac{6}{7}\cdot\frac{3}{8}$ without drawing a rectangle.

Practice problems…

$\frac{5}{9}\cdot\frac{2}{3}=$

$\frac{2}{5}\cdot\frac{2}{9}=$

When you have a problem with mixed numbers, the easiest way to do this computation is to change the mixed numbers to improper fractions. (You might think it would be easier to just multiply the whole numbers together, followed by multiplying the fractional parts together. Unfortunately, this doesn't work. If you want to understand why, do a YouTube search for "multiplying fractions with rectangles." There are several videos where you see why you actually have to do four separate products and add them all together. It ends up being much quicker to change mixed number to improper fractions, then treating them like the proper fractions above.)

$3\frac{3}{5}\cdot 2\frac{1}{12}=$

SIMPLIFYING BEFORE MULTIPLYING… Another way to do multiplication of fractions requiring reducing.

The $\frac{3}{4}\cdot\frac{5}{6}$ problem you did on the previous page has a product of $\frac{15}{24}$, but $\frac{15}{24}$ needs to be simplified since 15 and 24 have a common factor of 3.

$$\frac{3}{4}\cdot\frac{5}{6}=\frac{15\div 3}{24\div 3}=\frac{5}{8}$$

There are several ways to get to this simplified form of the answer. You can recognize that 15 and 24 have a common factor of 3 as shown above, or you can do a prime factorization of the numerator and denominator (like you did in the simplifying fractions activity in a previous unit.

$$\frac{3}{4}\cdot\frac{5}{6}=\frac{15}{24}=\frac{\cancel{3}\cdot 5}{2\cdot 2\cdot 2\cdot \cancel{3}}=\frac{5}{8}$$

But… since the original problem was a multiplication problem, we can SIMPLIFY BEFORE MULTIPLYING.

Notice that both the 3 (in the numerator) and 6 (in the denominator) in this problem have a common factor of 3. $\frac{3}{4}\cdot\frac{5}{6}$

This method will work with any MULTIPLICATION problem where a number in the numerator and a number in the denominator have a common factor.

Instead of multiplying first, then dividing out the common factor, we can divide out the common factor first, before multiplying.

$\frac{3}{4}\cdot\frac{5}{6}=\frac{\overset{1}{\cancel{3}}}{4}\cdot\frac{5}{\underset{2}{\cancel{6}}}$ Dividing 3 by 3 in the numerator gives 1 and dividing 6 by 3 in the denominator leaves 2.

So, $\frac{3}{4}\cdot\frac{5}{6}=\frac{\overset{1}{\cancel{3}}}{4}\cdot\frac{5}{\underset{2}{\cancel{6}}}=\frac{5}{8}$

Let's look at another problem, where this method may make more sense.

Multiply these two fractions. $\frac{21}{55}\cdot\frac{25}{28}$

If you don't simplify first, you get $\frac{21}{55}\cdot\frac{25}{28}=\frac{525}{1540}$ You should recognize that both the numerator and denominator are divisible by 5. This gets you to $\frac{21}{55}\cdot\frac{25}{28}=\frac{525\div 5}{1540\div 5}=\frac{105}{308}$, but then what? Chances are, you may not recognize that 105 and 308 are both divisible by 7, which would get you to $\frac{21}{55}\cdot\frac{25}{28}=\frac{525\div 5}{1540\div 5}=\frac{105\div 7}{308\div 7}=\frac{15}{44}$ which is the simplified answer.

However, if you simplify first, the problem can look like this…

$$\frac{\cancel{21}^{\,3}}{\cancel{55}_{\,11}}\cdot\frac{\cancel{25}^{\,5}}{\cancel{28}_{\,4}}=\frac{15}{44}$$

21 and 28 are both divisible by 7, and 25 and 55 are both divisible by 5.

This is much simpler than multiplying first then simplifying.

Try this problem.

$1\frac{1}{9}\cdot\frac{6}{14}\cdot\frac{21}{20}$ (Remember, you will want to change mixed numbers into improper fractions first.)

Did you get $\frac{1}{2}$ as an answer?

Here's another problem to try. $\frac{3}{5}\cdot\frac{10}{11}\cdot\frac{2}{9}$

Questions or confused? There is a video on the Multiplying Fractions page called Simplifying Before Multiplying that may help.

Practice multiplying these fractions.

$\frac{5}{8}\cdot\frac{6}{15}$

$4\frac{4}{5}\cdot 2\frac{1}{10}$

$\frac{2}{3}\cdot\frac{5}{8}\cdot\frac{6}{15}$

$\left(\frac{3}{4}\right)^2$

$\left(\frac{2}{3}\right)^3$

Remember that $\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\cdot\left(\frac{3}{4}\right)$