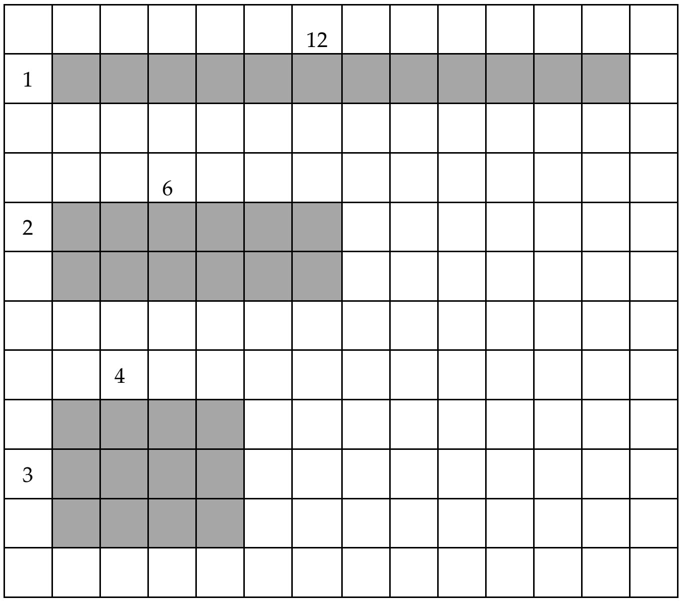
LISTING FACTORS

During the Exploring Rectangles activity, you listed factors by drawing all of the rectangles you could for a given number, then listing the dimensions of the rectangles.



There are three rectangles that can be made with 12 tiles or small squares. The dimensions of the rectangles are:

<u>12</u>
1 X 12
2 X 6
3 X 4

So the factors of 12 (remember to always list them from smallest to largest) are:

1, 2, 3, 4, 6, and 12

This method works great if the numbers are small or there aren't many rectangles, but what about a number like 72? Drawing all of the rectangles is cumbersome and time-consuming, but just trying to come up with the factors off the top of your head is prone to missing some.

This is a method that will guarantee being able to list ALL of the factors of any number. The first few times through, it may seem a little complicated and cumbersome, but it will get faster with practice.

Begin by writing the number 72 and underlining it, so you can remember which number you are finding the factors for. Then begin writing 1 X, 2 X, 3 X, beneath it.

<u>72</u> 1 X 2 X 3 X 4 X ⋮	As you list 1 X, 2 X, 3 X, etc. say (out loud or in your head) 1 X 1 =1 2 X 2 = 4 3 X 3 = 9 4 X 4 = 16 5 X 5 = 25 6 X 6 = 36 7 X 7 = 49 8 X 8 = 64 9 X 9 = 81 Whoa!!!! Notice that 81 is larger than the number 72 that we want to know the factors of. This means we don't need to write the 9 X.	So our problem should now look like… <u>72</u> 1 X 2 X 3 X 4 X 5 X 6 X 7 X 8 X

The process described in the middle column above guarantees we check high enough for factors, but no more than necessary. (If you are curious about why this works, think about how the rectangles drawn for 12 above start out long and skinny, then become more and more square like.)

Once we know which numbers we need to check, we can use the divisibility rules to check to see if they are factors.

One is always a factor of every number. 72 1 X 72 2 X 3 X 4 X 5 X 6 X 7 X 8 X	72 ends in 2, so it is divisible by 2. $72 \div 2 = 36$ 72 1 X 72 2 X 36 3 X 4 X 5 X 6 X 7 X 8 X	Remember the divisibility rule for 3. Since the digit sum of 72 is 9 ($7 + 2 = 9$), 72 is divisible by 3. 72 1 X 72 2 X 36 3 X 24 4 X 5 X 6 X 7 X 8 X
Remember the divisibility rule for 3. Since the digit sum of 72 is 9 ($7 + 2 = 9$), 72 is divisible by 3. 72 1 X 72 2 X 36 3 X 24 4 X 5 X 6 X 7 X 8 X	The divisibility rule for 4 isn't helpful for 2 digit numbers, but knowing that the answer to $72 \div 2$ is an even number (36 in this case), also tells me that 72 is divisible by 4. 72 1 X 72 2 X 36 3 X 24 4 X 18 5 X 6 X 7 X 8 X	72 isn't divisible by 5, because 72 doesn't end in a 0 or 5, so we just cross it out. 72 1 X 72 2 X 36 3 X 24 4 X 18 ~~5 X~~ 6 X 7 X 8 X

<table>
<tr>
<td>Since 72 was divisible by both 2 and 3, it must also be divisible by 6.
72
1 X 72
2 X 36
3 X 24
4 X 18
~~5 X~~
6 X 12
7 X
8 X</td>
<td>72 isn't divisible by 7. Remember that 7 X 10 is 70, and 7 X 11 is 77, so we can cross out 7 X.
72
1 X 72
2 X 36
3 X 24
4 X 18
~~5 X~~
6 X 12
~~7 X~~
8 X</td>
<td>And finally, 72 is divisible by 8. Since 4 divided into 72 an even number of times, 8 must divide into 72 without a remainder.
72
1 X 72
2 X 36
3 X 24
4 X 18
~~5 X~~
6 X 12
~~7 X~~
8 X 9</td>
</tr>
</table>

So the factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

It might seem like this is a pretty tedious process, but it is efficient for finding all of the factors of a number. Here's another example…

Find all of the factors of 221.

It might seem like a big number like 221 will take forever to do, but not so…

<table>
<tr>
<td>Remember the first step is to list all of the possible first factors.
221
1 X
2 X
3 X
4 X
5 X
6 X
7 X
8 X
9 X
10 X
(remember that we have to keep going because 10 X 10 is only 100)
11 X
12 X
13 X
14 X
Since 14 X 14 = 196 and 15 X 15 = 225 (which is bigger than 221) we only need to check through 14.</td>
<td>1 is a factor of every number.
221
1 X 221
2 X
3 X
4 X
5 X
6 X
7 X
8 X
9 X
10 X
11 X
12 X
13 X
14 X</td>
<td>Since 221 is not divisible by 2, we can not only cross out 2, but we can cross out all of the multiples of 2.
221
1 X 221
~~2 X~~
3 X
~~4 X~~
5 X
~~6 X~~
7 X
~~8 X~~
9 X
~~10 X~~
11 X
~~12 X~~
13 X
~~14 X~~

We are half way done!</td>
</tr>
</table>

Since the digit sum of 221 is 5 (2 + 2 + 1 = 5), 221 is NOT divisible by 3. This also means that 221 will not be divisible by 6, 9, 12… by any multiple of 3. 221 1 X 221 ~~2 X~~ ~~3 X~~ ~~4 X~~ 5 X ~~6 X~~ 7 X ~~8 X~~ ~~9 X~~ ~~10 X~~ 11 X ~~12 X~~ 13 X ~~14 X~~	221 isn't divisible by 5 (or 10). 221 1 X 221 ~~2 X~~ ~~3 X~~ ~~4 X~~ ~~5 X~~ ~~6 X~~ 7 X ~~8 X~~ ~~9 X~~ ~~10 X~~ 11 X ~~12 X~~ 13 X ~~14 X~~	221 isn't divisible by 7 or 11 (you might want to start checking these with a calculator), but it is divisible by 13. 221 1 X 221 ~~2 X~~ ~~3 X~~ ~~4 X~~ ~~5 X~~ ~~6 X~~ ~~7 X~~ ~~8 X~~ ~~9 X~~ ~~10 X~~ ~~11 X~~ ~~12 X~~ 13 X 17 ~~14 X~~

Most people don't know that 221 is divisible by 13 off the top of their head, and it would be really easy to miss it unless you have a systematic way to check for factors and you know how high you need to check. So, even though 221 may look prime, it isn't. The factors of 221 are 1, 13, 17 and 221.

Practice the processes described above to find all of the factors of the following numbers.

81	132	57
91	60	97

Remember that numbers that have only two factors (1 and the number itself) are PRIME.